ABSTRACT. A shift in mathematics education in the Netherlands towards the so-called realistic approach made it necessary to prepare prospective teachers for a type of curriculum different from what they experienced as pupils. This article describes the characteristics of a preservice programme aiming at this goal and presents an analysis of the development of the student teachers’ views of mathematics and mathematics education during the programme as well as their classroom behaviour. This analysis is based on two research studies. The first was a longitudinal study in which the student teachers were followed during 4½ years by means of questionnaires and interviews. The second was a study in which graduates from this programme were compared with graduates from a more traditional preparation programme by means of two teacher questionnaires and a pupil questionnaire, the latter measuring the pupils’ perceptions of the actual teaching behaviour of the graduates.

The teacher education programme appeared to be successful in changing the student teachers’ views of mathematics education, especially in the direction of a more inquiry oriented approach, and in promoting effective teacher behaviour in the classroom. As far as their facilitating role as a teacher is concerned, the student teachers seemed to go through a two-stage learning process. Most of them reached the first stage, in which they realize that pupils have different preferences for learning and that a variety of possible explanations for problems should be offered. However, only a small number of student teachers seem to reach the second stage, in which they recognize the principle of building on pupils’ own constructions, an important feature of realistic mathematics education. Possible explanations for the low impact of the programme, as well as solutions are discussed.

1. INTRODUCTION

Over the last twenty years mathematics education in the Netherlands has changed considerably, shifting from a mechanistic and sometimes structuralistic to a so-called ‘realistic’ approach, both in primary and secondary education. According to the mechanistic point of view, mathematics is a system of rules and algorithms. The emphasis is on verifying and applying these rules to problems that are similar to previous problems. Much attention is paid to a careful stepwise approach, memorizing, and learning the ‘tricks’. In the structuralistic view (e.g., Dieudonné, 1982), mathematics (as a cognitive achievement) is an organized, deductive system and the

* We would like to thank three anonymous reviewers and the editor for their helpful comments on an earlier version of this manuscript.

learning process in mathematics education should be guided by the structure of this system. Realistic mathematics education in the Netherlands starts from a completely different point of view and aims at the construction by children of their own mathematical knowledge by giving meaning to problems from a real world context (Freudenthal, 1978; Treffers, 1987). Pupils are challenged to develop their own strategies for solving such problems, and to discuss these with other pupils. Finding solutions to real-world problems is not the end of mathematics lessons in this approach. Teachers help the children to develop their informal strategies into more formal approaches which they can then use in other situations (Treffers, 1987).

The main difference with the mechanistic and structuralistic approaches is that realistic mathematics education does not start from abstract principles or rules with the aim to learn to apply these in concrete situations, nor does it focus on an instrumental type of knowledge. The process of constructing knowledge and principles by the pupils themselves gets the main emphasis. As Freudenthal (1978) puts it, this reflects a shift from mathematics as a created subject (created by others, especially mathematicians) towards mathematics as a subject to be created. This implies a more dynamic view of mathematics: mathematical actions and the process of developing strategies gets much more attention in comparison with static clear-and-cut knowledge. Freudenthal (1991) emphasizes that this presents children with the opportunity to ‘reinvent’ mathematical ideas.

A typical example of a realistic mathematics problem is keeping track of ongoing changes in the number of bus passengers (Gravemeijer, 1994). In the situation of passengers who enter and leave a bus, addition and subtraction emerge in a natural way. The narrative about what happens at every bus stop leads to the development of some sort of-representation of the changes in the number of passengers (e.g. the description in Figure 1).

It would be in line with the basic ideas of realistic mathematics education if the pupils were allowed to discuss the problems in small groups, and stimulated to develop their own informal approaches. In a follow-up discussion facilitated by questions from the teacher the pupils’ reflection on their strategies would then be promoted and steps made towards more formalization of the useful approaches. During this process the representation of Figure 1 can develop into the more formalized language of Figure 2.
Finally standard forms of notation \((2 + 4 = 6)\) emerge, after working with situations involving static comparison (Van den Brink, 1984).

In this view of mathematics education a problem like ‘do the medians of a triangle intersect at a single point’ would not be considered as a realistic problem. Although it is concrete, a context is missing that they recognize from experiences in everyday life. On the other hand, not every contextualised problem is a realistic problem. The criterion for a problem to be called realistic is that it should be likely that the problem is experienced by the learner as real and personally interesting. This is often the case in problems that arise from situations in which the pupils work with hands-on materials, which implies that realistic problems need not always originate from the pupils’ everyday life.

In realistic mathematics education formalization is reached in several consecutive stages. Therefore a problem such as presented above only illustrates a part of realistic mathematics education. In order to understand the principles more fully a set of curriculum materials for at least a month should be described (e.g. De Lange, 1987). Such a description, which would include how the particular mathematical knowledge needed for the solution of a real life problem is acquired by students, is clearly outside the scope of this article. For more information about the development towards realistic mathematics education we refer the reader to vol. 25, no. 1–2 of this journal, entitled ‘the legacy of Hans Freudenthal’.

A consequence of the ‘realistic turn’ in mathematics education was that teachers had to be prepared for a curriculum with very distinct features from the curriculum they experienced themselves when they were pupils in school. Although empirical data on traditional mathematics classroom practices in the Netherlands are scarce, there are data which reflect the highly didactic nature of these practices. Tomic (1985), for example, showed that pupils in secondary school mathematics spent only 11% of the instruction time in talking with the teacher, 22% of the time they were doing independent seat work, and 51% of the time they were listening or homework was being reviewed. In interviews with the student teachers in the research study discussed in this article, eight out of ten student teachers were found to have experienced extremely teacher-oriented mathematics teaching in high school (at the end of the seventies and the beginning of the eighties): most of the time the teacher was talking or explaining something,
while the pupils were copying from the blackboard. If the teacher was not
talking, the pupils were involved in independent problem solving. Usually
these problems were analogous to problems the teacher had already solved.
This is reflected in two examples of statements by these student teachers:

I used to look up to my mathematics teacher (high school), as if he were some kind of ideal. But I don’t do that any more. He was someone who taught mathematics the way they all used to do: sums on the blackboard, an example, next kind of sum, make your own example, and then more sums.

In high school you’re used to having the teacher figure it all out and then pass it on to you.

Many researchers claim that the way in which student teachers were
taught a subject in school has a strong influence on their conceptions
about that subject and their conceptions about learning and teaching it
(Kagan, 1992; Lortie, 1975; Thompson, 1984; Huibregtse, Korthagen &
Wubbels, 1994). Stofflett and Stoddart (1994), for example, have shown
that student teachers who themselves experienced learning in an active
way, are (according to their own reports) more inclined to plan lessons in
a way that facilitates active knowledge construction. As these researchers
stress, this creates the challenging task for teacher education to break the
didactic teaching-learning-teaching cycle. When, during the seventies and
eighties, realistic education in the Netherlands was being developed, this
task was given high priority in a programme in Utrecht for the training
of mathematics teachers in secondary education. Until recently, this was
a $4\frac{1}{2}$-year programme, in which student teachers had to take a second
subject in addition to mathematics. A total of one year, distributed over
the $4\frac{1}{2}$-year period, was devoted to the professional preparation. The same
staff was responsible for both the mathematics and the professional part
of the programme. Evaluative research has been done on this teacher
education programme, as it was one of the very first that builded on the
principle of promoting reflection, both in the mathematics component and
the professional preparation (see for example Korthagen, 1985, 1988 and
Korthagen & Wubbels, 1995). Although the programme on which this
research focused, has changed since, the research results still seem to be
important for other attempts in teacher education to promote changes in
teacher’s conceptions about mathematics and the learning and teaching of
mathematics.

In this article we focus on the development of the student teachers’
conceptions of mathematics and mathematics education. First we present
in section 2 more information about the programme. Section 3 describes
the design of two studies into the programme. The research results of a
longitudinal study are presented in section 4 and of a comparison study in
section 5. Finally, in section 6 we discuss these results and we put forward suggestions for improving mathematics teacher education.

2. THE TEACHER EDUCATION PROGRAMME

In this section we discuss the main characteristics of the Utrecht teacher education programme.

Reflection
As Korthagen (1985, 1988) describes, the promotion of reflection was a major goal in this programme. For the professional preparation this meant that a lot of emphasis was put on the student teachers’ capacity to analyze their own teaching. This capacity was considered as an important means for directing one’s own professional development. The aim was to produce teachers who would ultimately be capable of independently tracing a process consisting of the following phases:

1. Action: confrontation with a concrete and real situation which requires action;
2. Looking back on the situation and the action;
3. Awareness of essential aspects;
4. Creation of alternative solutions or methods of action;
5. Trial.

Phase 5 also forms the first phase of a new cycle. This is a spiral model, known as the ALACT model, after the initial letters of the five phases (Korthagen, 1985). The crux of the ALACT model lies in phase 3, where a mental structure is formed, or an existing mental structure altered. Therefore, the following definition given by Wubbels and Korthagen (1990) characterizes the conceptualization of reflection in the teacher education programme: ‘reflection is the mental process of structuring or restructuring an experience, a problem or existing knowledge or insights.’

The first phase of the ALACT model is an action, and thus reflection was always based on actual experiences of the student teachers in concrete situations. In the professional component of the programme, reflection was concerned first of all with the student teacher’s own cognitions, thinking processes, feelings, aims and behaviour in interaction situations with their fellow students, for example during their collaborative work on the course materials. In the next stage of the programme the student teachers were stimulated to broaden their reflections to also include the thinking and feeling of these fellow students in such interaction situations. Gradually the focus of the reflection shifted towards the interaction with pupils in
the classroom. The final goal was that the student teachers learned to proceed through the spiral model without the help of the supervising teacher educators and learned to become independent learners who were able to direct their own professional development. This involves reflection by the student teachers on their own learning processes.

This development in the focus of reflection was paralleled by a similar development in the mathematics component of the programme. At first, reflection focused on the personal process of working on mathematical problems and learning mathematical concepts. At regular intervals, the student teachers were asked to hand in written reports on the way they worked on particular mathematical problems, regardless of whether they had found a solution to the problem or not. In this way reflection on the process of ‘doing’ mathematics was emphasized. Then reflection shifted to the mathematical processes involved in working on problems in groups. The ultimate object of reflection was the process of helping pupils in secondary school to learn mathematics.

The ALACT model had its analog in the mathematics component of the programme. Mathematical actions and the real world were taken as starting points for reflection. The teacher educators wanted the student teachers to develop mathematical structures by reflecting on problems taken from reality, and on their own constructive steps in trying to solve these problems. In this context phase 4 of the ALACT model means creating a solution or solutions to a mathematical problem and phase 5 the confrontation of the solution(s) with the real world problem. Figure 3 clarifies this ‘adaptation’ of the ALACT model to action in and reflection on situations in which mathematics can be used and developed.

An inquiry-oriented approach

In line with the professional preparation part of the programme, in which the student teachers were stimulated towards inquiry into their own teaching experiences, the mathematical component of the programme was highly inquiry-oriented. The inquiry process is characterized by the consecutive steps of translating a real world problem into a mathematical problem, the analysis and structuring of such a problem, the creation of a mathematical solution, the translation of this solution to the real world and the reflection on the merits and restrictions of the solution, which can be followed by a next cycle in which the translation of the problem is refined, generalized or otherwise changed.

For many student teachers this strong emphasis on inquiry meant a big change from what they were used to in secondary education. This required
a careful strategy of gradualness in order to give them sufficient structure and support. Pratt (1988) calls this ‘staged withdrawal’.

As regards the student teachers’ reflection on their working and learning processes, the programme did away with the idea that mathematics is primarily a mental exercise. Feelings and attitudes quite naturally came to the fore in such aspects as the fun of problem-solving, an aversion to a particular problem, the experience of sinking one’s teeth into a problem, the pleasure of working together and the excitement when something finally ‘dawns’. In this way the educators tried to make the prospective teachers aware of the significance of the affective aspects of the learning and inquiry process.

**Using mathematics in real life contexts**

In accordance with the view of realistic mathematics education described above, the teacher educators believed that mathematical concepts should be developed on the basis of problems present in real-life contexts, and that mathematics should be useful in everyday situations. The course materials mainly consisted of real and concrete problems, which the students
would tackle by means of analyzing, structuring and testing alternative solutions. Problems in real-life contexts and problems from other domains than mathematics occupied a prominent place in most of the courses. Several examples are presented in appendix 1.

Many of the problems presented to the student teachers confronted them with the fact that many problems do not necessarily have only one correct answer, as many of them were led to believe during their high school mathematics classes. In real life most problems have no simple solution or many possible solutions. This makes it more interesting to compare solutions and the ways in which the various students tackled the problem.

Mathematics as an activity of individuals and groups
The mathematics in the programme was often linked to hands-on activities. In courses in the professional component pedagogical theories were discussed emphasizing the importance of hands-on activities in learning. The student teachers’ own experiences in the mathematics courses were taken as the basis for these discussions.

The emphasis on mathematical activities was to be found both in the individual work of student teachers and in group assignments. Collaboration with fellow students in the early stages of the programme was encouraged, in order to illustrate that mathematics does not always mean following straightforward procedures, and in order to provide practice in helping others and explaining problems. Gradually, in the course of the first two years of the programme, the emphasis shifted to helping individual pupils in high school, then to groups of pupils and finally to whole classes. The importance of pupils’ activities in the mathematics classroom was emphasized in the supervision of teaching practice and the student teachers were helped to find ways for activating pupils in cases in which the text books at school did not offer many opportunities.

The teacher’s role: facilitating learning
One consequence of many of the principles mentioned above is that the teacher’s role is one of facilitating learning, rather than transmitting knowledge. In the programme teacher educators served as the role models for this view of the teacher. Throughout most of the programme, very few of their classes took the form of lectures; rather they were organized around individual or group activities, and individual or group supervision. The teacher educators strived to initiate discussion, rewarded initiative and creativity, and worked with the student teachers in a cooperative way. According to Treffers (1987), in realistic mathematics education teachers should use pupils’ constructions to help them build new constructions, solve problems,
etc. For the teacher educators this was an essential part of their facilitating role in working with the student teachers.

3. DESIGN OF THE RESEARCH STUDIES

The first study about the teacher education college was initiated in 1984 and will be referred to as the longitudinal study. A group of 18 students from the teacher education programme were followed during the four and a half years of the programme, by means of questionnaires, interviews and video recordings of supervision conferences (for details of this study see Korthagen, 1988). Of the 18 student teachers in the group, eight left before the end of the second year. One reason for the decision to terminate their studies seemed to be that the emphasis on reflection in the programme did not fit in with their own learning style. In this article the interview data and the scores on a questionnaire dealing with realistic education refer only to the remaining ten students who stayed in the programme for four and a half year. In the interviews the strategy used was intended to elicit conscious ideas, experiences and opinions. For this reason we used open-ended questions (e.g. ‘What views on mathematics education do your teacher educators have?’), rather than asking whether they agreed or disagreed with the statements of the interviewer (‘Do you think that your teacher educators want mathematics education to be useful in real life?’). The interviews were analyzed using an inductive strategy (constant comparison; Glaser & Strauss, 1967).

A second study was carried out in which graduates of the teacher education programme were compared with graduates of another programme, in which a more traditional view of mathematics education was presented (the comparison study; Wubbels & Korthagen, 1990). The graduates of the programmes had been teaching between one and ten years at the time of the study. They were compared by means of several instruments, two of which are relevant in the framework of this article. The first was an instrument devised by Korthagen (1988) to measure reflective attitude within the domain of mathematics. The second was a pupil and teacher questionnaire developed by Wubbels et al. (see Wubbels & Levy, 1993) and was administered to the teachers and pupils of two of their classes. This instrument, the Questionnaire on Teacher Interaction (QTI) maps the teacher’s and pupils’ perceptions of the interpersonal teacher behaviour.

The studies were not originally designed for the purpose for which they are used in this article. As a result it is not possible to draw firm conclusions from the material about the specific relationships between certain programme elements and the effect of the programme. Differences
between graduates from different programmes could be due to attrition from the programme rather than to the impact of programme elements.

4. RESULTS OF THE LONGITUDINAL STUDY

4.1. Attitude towards realistic mathematics education

At the end of the programme student teachers were asked to complete a questionnaire about their opinions on the aims and methods of good mathematics education. One scale, consisting of five items, was designed to measure opinions on the desirability of realistic mathematics education ($\alpha$-reliability = 0.85) A sample item is: ‘In the choice of topics the teacher should first of all build on the experiential world of the pupils’. The same questionnaire was administered to a sample of mathematics teachers at a meeting about the grading of mathematics examinations. Their average number of years of teaching experience was 15.7 (st.dev. 8.2). The graduates of the programme are significantly more in favour of realistic mathematics education than their more experienced colleagues (mean scores respectively 3.8 ($n=10$) and 3.4 ($n=66$); $t$-value 1.7; $p<0.05$). The effect size is 0.6 standard deviation which is considered a medium effect in terms of Cohen (1988). We consider this result as a confirmation that the graduates of the programme are inclined to be in favour of a more realistic approach to mathematics education.

4.2. Programme characteristics

The studies about the programme show that many of the student teachers recognize the characteristics mentioned in the previous sections. We illustrate this with some quotations from the interviews. In section 4.3 we will discuss the number of student teachers who recognized various characteristics, and their willingness to choose the same starting points for their own teaching of mathematics in secondary education.

In high school the teacher stands at the front of the class, and everyone does what he says. All you have to do is pay attention and do your homework, and there’s no problem. Here it was more like: here’s your problem sheet, see how you make out. They didn’t give you any formulas or anything. You had to figure them out for yourself.

In the beginning especially, in the first and second year, I think it’s awfully important that they [pupils] enjoy math. I want them to suddenly discover - hey, this has something to do with the real world out there.

An awful lot of it you have to do yourself. You have to work it out for yourself, and sometimes that means spending a lot of time on a problem. But most of the time it works. In high school certain things were explained, and if you understood it all, then you could go on doing that kind of problem indefinitely.
Here you have to work everything out for yourself. You’re in a group of three or four people, and if you can’t figure it out together, then you have to spend half an hour or so on your own until you get it.

One thing I noticed in the ‘Using Math’ class was that if you got off on the wrong track, you shouldn’t think ‘what a waste of time’, because it might have brought to light interesting things you wouldn’t otherwise have seen. You worked in small groups and everyone tackled one part of the problem. Then you got together with the teacher and everyone had to show what they’d done and how far they’d gotten. If you’d started off wrong, you might say, ‘I was working on it but I ended up with some crazy answer, so I left it at home.’ Then the teacher would say, ‘I want to see it anyway - that’s just the sort of thing I’m interested in. Even a wrong solution can produce interesting information.’ I think that’s a really positive approach. In high school it was right or wrong, and then you went on to something else. But now I see that it’s important for you to figure out where you went wrong, or why you went off in a direction that didn’t lead anywhere.

It is obvious from these statements that the manner in which mathematics is taught in the teacher education programme differed significantly from what the student teachers had experienced in high school. Statements about the inquiry approach, about mathematics as an activity of groups and individuals, about using mathematics and the use of real world problems are present in these excerpts. We will illustrate later on that student teachers also recognize what role of the teacher is promoted by the programme. On two points however the programme seems to be less successful than the teacher educator staff wanted: solving real life problems with mathematics and the role of the teacher as a facilitator of learning.

_Solving real-life problems with mathematics_

In the interviews student teachers often say that real-life problems were an important part of the courses in their teacher education programme. They also mention that it is important to bring real-life problems into the high school classroom. An example:

Interviewer: Working together with others, thinking about yourself - what else do your teacher educators consider important? Student teacher; They want you to learn how to apply mathematics, so that you know what it’s for, and can teach the same way later on. And so you can show how it can be used.

As a rule, these statements refer to an application context. This means that the student teachers are aware of one of the steps in the process of solving a real life problem with the help of mathematics (Figure 3): using the mathematical results in the real world, or translating the mathematical results back to real-life problems. Less frequently, the student teachers say in the interviews that real-life problems should be used as contexts from which to initiate the use of mathematics. This is the first step in the process. The following excerpt from an interview is one of the few instances where a student teacher mentions this first step.
During our course you’re forced to make all the calculations concrete, by thinking up situations from the everyday world. The kids at technical school always want to know what good it is to them, and all.

Some student teachers mention that extracting mathematical problems from real-life situations was a characteristic of the programme, but that they doubt if they will be able to do the same kind of thing in their own classroom teaching.

The interview data lead us to conclude that the programme did not succeed in making all the student teachers think about the problem solving process according to the model presented in Figure 3, let alone that they would use the model in their teaching. From their statements we infer that they will include in their teaching the component inside the mathematical model (the creation of a mathematical solution) and the translation back to the real world. Certain important aspects were missing from the statements of some student teachers, such as finding mathematics in contexts, and reflection on the merits and restrictions of the mathematical model, and on the merits and restrictions of the results in the real world.

The role of the teacher

From the data that we collected in the longitudinal study, it appears that the teacher education programme is successful in helping prospective teachers to shape their role according to the principles of realistic mathematics education. Student teachers seem to go through a two-stage learning process. After the first stage, the student teachers mention that pupils may have different preferences for learning, and that for this reason several possible explanations should be offered to them. However, they express views that still represent a ‘teaching-as-transmission’ standpoint. When student teachers reach the second, more pupil-centred and constructivist stage, they mention the fact that a teacher needs to find out how pupils think and learn, in order to be able to build on what they already know. Although some of the student teachers do reach the second stage, others do not get beyond the first stage. The main thing that the latter student teachers seem to learn from the programme is that they have to present several different explanations and examples. Here are two examples of student teachers who have reached the first stage (still; teacher-centred):

(interview in fifth year of the program)

I now have an entirely new outlook on math. When I’d finally grasped some mathematical point I used to think ‘OK, that’s the way it works’. Now I have to get it across to a whole group of kids who all have to see ‘how it works’. So now, instead of just accepting something once I understand it, I start looking for new ways of making it clear to someone else.
If I have a preference for a particular method of explaining something, then I tend to use that method all the time. But maybe the kids would rather have me use another method. I’m always trying to think up mnemonics for them, things that make it easy to remember something. Yesterday there was a boy in my class who had a question, and all of a sudden I thought, ‘I’ve got to explain this in two different ways, because the method I usually use is probably going to be too difficult for him, even though he can see that it’s right. After that I can explain it the other way.’ So I found myself trying to use several different methods.

Often the first result of the programme is that student teachers want to make lessons more attractive, want to give more examples and offer various ways of solving problems. However, this does not mean that they are trying to build on the pupils’ constructions. Student teachers who reach the second stage mention that it is important to try to think as the pupils do; they want to use this knowledge in their explanations, or in helping their pupils to actively develop their constructions. A year later (in the fourth year of the program), the second student teacher above seems to have adopted the ideas about using pupil constructions to the level of the second stage:

How to explain things, like all those complicated mathematical problems. And when you do it for certain problems, it’s easier to see how you could use it for other problems too, and then you start seeing things in a broader perspective. I think I’ve learned that first you start with the knowledge that your pupils already have. Taking that as your starting point, you try to figure out how to go on to something they don’t know yet. And once you’ve tried this out with a few subjects, then you start to see the broader outlines, a kind of system, and you do the same thing with the next problem. This way it’s easier to step back and see mathematics as a whole, than if you just sat down and tried to figure it out. I realize that I’ve learned a lot from this myself. Now if I have to explain something to someone, I’ve already developed a kind of strategy.

It is clear that this student teacher’s views on teaching have changed, from explaining to facilitating learning and helping to analyze. The next example also concerns a student teacher who actively wants to take pupils constructions seriously.

Yes. I used to help my little brother with his sums when I was in high school, and I’d say ‘this is the way you do it’, and then I’d do the whole thing for him. Whereas now I say something like ‘how did you try to solve it’, and then I go over it to see where he went wrong, and why. But I’m not doing the whole sum for him. I’m making him stop and think about what he’s doing, but he’s actually doing it himself.

However, this student teacher is one of the very few exceptions. One could wonder whether the teacher educators in the programme did clearly model building on the learners’ constructions. From the interviews, however, it appears that this is definitely the case:

1: What kind of ‘message’ do you think the course wants you to take away with you?
S: A lot of different methods of approaching a problem. So that you can explain it in several different ways. That way you aren’t limited to your own patterns of thought, you also have some idea of how other people’s minds work.

I: What’s so different about what you learn here?

S: Take the course on subject matter pedagogy, for instance. First you look to see how much you already know about a particular subject, how you were taught yourself. And then you go on from there. You think about the way you learn something now, and the way you used to learn.

However, the question remains how many student teachers recognize the principle of building on the learners’ constructions as being important. We deal with this question in the next section.

4.3. Programme characteristics and views on mathematics education

In the interviews held during the longitudinal study, the student teachers were asked to put into words just what it was that was different in the way that mathematics was taught at the teacher education college compared to their secondary education. Table 1 shows the number of student teachers who mentioned the particular characteristics discussed above. According to the table, two of the four characteristics (the inquiry approach and mathematics as an activity) were apparently so obviously present that they were mentioned by all the student teachers. The characteristic ‘starting from context problems’ was mentioned by more than half of the student teachers. We distinguished between two aspects of the facilitating role of the teacher; one aspect ‘using different explanations’ was mentioned by almost every student teacher, the other ‘building on pupil constructions’ by very few.

On the basis of the interviews with the student teachers, it appears that most of them drastically altered their views on mathematics education in the course of the programme. For most of them this change took place very quickly, during the first year of teacher education. There were others, however, whose views changed more gradually. Using the descriptions of their lessons given in the interviews, we were able to establish for eight of the student teachers what kind of mathematics teaching they themselves wanted to provide in the classroom. As Table 1 shows, the inquiry approach is mentioned by nearly all the student teachers. Using different explanations as part of the facilitating role of the teacher is also widely seen as a good way of teaching. There is little explicit support among the student teachers for the other aspects of realistic mathematics education, in particular for the building on pupil constructions as part of the facilitating role of the teacher. A representative statement of a student teacher who reached the first but not the second stage is the following. It is an excerpt from an interview held towards the end of the programme.
TABLE I

Number of student teachers who identified a specific characteristic of the teacher education program, and number of student teachers who think a specific characteristic is important in high school mathematics education (n = 10).

<table>
<thead>
<tr>
<th>Present in teacher education program?</th>
<th>Important in high school?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Using mathematics in contexts</td>
<td>8</td>
</tr>
<tr>
<td>2. Inquiry approach</td>
<td>9</td>
</tr>
<tr>
<td>3. Mathematics as an activity</td>
<td>9</td>
</tr>
<tr>
<td>4. Teacher in facilitating role</td>
<td></td>
</tr>
<tr>
<td>a) using different explanations</td>
<td>9</td>
</tr>
<tr>
<td>b) building on pupil constructions</td>
<td>3</td>
</tr>
<tr>
<td>Total number of students who</td>
<td>9</td>
</tr>
<tr>
<td>mentioned issues in this area</td>
<td></td>
</tr>
</tbody>
</table>

Everyone prefers a different way of being taught. Some like it inductive, others deductive. Everyone is very different in that respect, the pupils, too, I think. I have always been taught in a way that you learned the trick, and now I realize much more that understanding is also very important. And I would explain things very differently now from how I did it in the first year of the programme, so in this respect I changed very much.

Due to the open interview and questionnaire strategy used (see section 3), the results of the longitudinal study may underestimate the number of student teachers that adhere to certain opinions.

5. RESULTS OF THE COMPARISON STUDY

5.1. Reflective attitude towards mathematics

In the study in which we compared graduates of the Utrecht programme with graduates from another Dutch programme, the reflective attitude towards mathematics was measured by means of a scale taken from the IEO-test (Korthagen & Verkuyl, 1987). It has five items, to be answered on a five-point scale ranging from ‘strongly disagree’ (1) to ‘strongly agree’ (5). A sample item is ‘After a mathematics class I find myself thinking about mathematical problems that came up in class’. The $\alpha$-reliability was 0.80. It appears that the attitude of the graduates of the Utrecht programme as regards mathematics is not significantly more reflective than that of graduates of a programme of another teacher education college that presented mathematics more as a static structure (mean score respectively 3.8 and 3.6; $t$-value 0.9). This result is not in line with our expectations. We
thought it plausible that as a result of the emphasis on the process character of mathematics in this programme, and the inquiry oriented approach, the Utrecht graduates would have a more reflective attitude. Apart from a real lack of influence of the programme on the student teachers in this respect another explanation of this result is possible. The graduates of the Utrecht programme may have higher standards for reflection than the graduates of the other programme. The Utrecht programme may have made the student teachers aware of what inquiry and reflection can mean and may make them therefore cautious to state easily that they engage in a lot of reflection. The graduates of the other programme may consider themselves inquiring easier because they have experienced less what inquiring activity can mean. This explanation points to a threat to the validity of the questionnaire methodology employed. Therefore studies using more extensive interviewing and perhaps think aloud sessions are needed.

5.2. Interpersonal teacher behaviour

All the previous results touch on characteristics of the cognitions of student teachers and graduates of the programme. It is well known that these cognitions do not say very much about the actual behaviour in classroom. Therefore we present also the results of an investigation into the teacher and pupils perceptions of the interpersonal teacher behaviour. To compare the pupils’ perceptions of different groups of teachers information was obtained from the pupils in two classes of every teacher with the Questionnaire on Teacher Interaction (QTI). The QTI consists of 77 items that map teacher behaviour in eight scales called, leading, helpful/friendly, understanding, giving pupils responsibility and freedom, uncertain, dissatisfied, admonishing and strict. The QTI has been used extensively for research purposes and has proved consistently reliable (see Wubbels & Levy, 1993). Here too, the internal consistencies of all the QTI scales were above 0.80. We compare the graduates from the Utrecht and the other programme on the quality of their behaviour (as perceived by the pupils) and on the quality of the self-perception of their behaviour.

Quality of the teacher behaviour

Research of Wubbels, Brekelmans and Hooymayers (1991) showed that the average pupil scores in a class on the scales leading, helpful/friendly and understanding are positively correlated to both cognitive and affective pupil outcomes: the more pupils see a teacher as leading, friendly or understanding, the higher they score on a cognitive test and the more positive is their opinion of the lessons and the subject. Wubbels et al. (1991) also report that scores for uncertain, dissatisfied and admonishing teacher
behaviour display a negative correlation with cognitive and affective outcomes. It is possible to derive from this a criterion for the quality of teacher behaviour. According to this criterion, the higher pupils rate teachers on those scales which are positively correlated with pupil outcomes and the lower they rate them on the negatively correlated scales, the better the interpersonal teacher behaviour is.

For each class we determined the average pupil score on the eight scales. The scores of each teacher’s two classes were then averaged. In this way eight scale scores per teacher were obtained, to serve as a measure of pupil perceptions of teacher behaviour.

**TABLE II**

Pupil perceptions of teacher behaviour in the two groups. The same information for teachers one or two years after graduation and three or more years later.

<table>
<thead>
<tr>
<th>Scale</th>
<th>All teachers</th>
<th>Teachers 1 and 2 years after graduation</th>
<th>Teachers 3 or more years after graduation</th>
</tr>
</thead>
<tbody>
<tr>
<td>n = 37</td>
<td>n = 36</td>
<td>n = 7</td>
<td>n = 9</td>
</tr>
<tr>
<td>Leading</td>
<td>3.6</td>
<td>3.4</td>
<td>3.4</td>
</tr>
<tr>
<td>Helpful/friendly</td>
<td>3.9</td>
<td>3.6</td>
<td>3.9</td>
</tr>
<tr>
<td>Understanding</td>
<td>4.0</td>
<td>3.7</td>
<td>3.9</td>
</tr>
<tr>
<td>Giving responsibility</td>
<td>2.8</td>
<td>2.7</td>
<td>2.9</td>
</tr>
<tr>
<td>Uncertain</td>
<td>2.0</td>
<td>2.1</td>
<td>2.2</td>
</tr>
<tr>
<td>Dissatisfied</td>
<td>2.3</td>
<td>2.4</td>
<td>2.5</td>
</tr>
<tr>
<td>Admonishing</td>
<td>2.7</td>
<td>2.8</td>
<td>2.6</td>
</tr>
<tr>
<td>Strict</td>
<td>2.7</td>
<td>2.7</td>
<td>2.7</td>
</tr>
</tbody>
</table>

1 All scale scores have been transformed so that they have a range of 1.0 to 5.0, to coincide with the five-point scale provided for each item: 1 = strong disagreement; 5 = strong agreement.

Table 2 shows the average pupil scores on the eight QTI scales in the two groups of teachers. The table shows that the pupil scores for the Utrecht group are higher than those for the other group on all the scales which are positively linked to cognitive and affective outcomes. For all the scales which are negatively related to the outcomes the Utrecht group is perceived lower than the other group. In order to be able to test differences between the two groups on the pupil perception criterion with one test, a global measure for the quality of the pupil-teacher relationship was calculated: The leading, helpful/friendly and understanding scores were summed up.
and then the uncertain, dissatisfied and admonishing scores were subtracted from this sum.

### TABLE III

Global quality score in the two groups and the results of a \( t \)-test on the group means.

<table>
<thead>
<tr>
<th></th>
<th>Utrecht group</th>
<th>Other group</th>
<th>( t )-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>( n )</td>
<td>( n )</td>
<td></td>
</tr>
<tr>
<td>quality</td>
<td>quality</td>
<td></td>
<td></td>
</tr>
<tr>
<td>All teachers</td>
<td>37 4.5</td>
<td>36 3.5</td>
<td>1.9*</td>
</tr>
<tr>
<td>Teachers 1 &amp; 2 years</td>
<td>7 3.0</td>
<td>9 0.4</td>
<td>-0.3</td>
</tr>
<tr>
<td>after graduation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teachers 3 or more</td>
<td>30 4.9</td>
<td>27 3.6</td>
<td>2.2*</td>
</tr>
<tr>
<td>years after graduation</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* \( p < 0.05 \)

\(^{1}\) This analysis could only be performed on class-level. The other analyses have been carried out after taking the mean scores of the two classes of a teacher.

In Table 3 we show this global quality score and the results of a \( t \)-test on the group means of these scores. The mean score in the Utrecht group is significantly higher than in the other group; the effect size is 0.6 standard deviation, which may be seen as a medium effect (Cohen, 1988).

Analyses for the more recent graduates (less than three years before) show that in the first two years of professional experience there are no significant differences in the pupil perceptions of the interpersonal behaviour of the two groups. In the case of those who graduated some time before, however, the difference is significant. From Table 2 one can see that the direction of the differences is the same as in the group of teachers as a whole. Thus the better teacher behaviour of the graduates of the Utrecht programme would seem to be a long-term rather than a short-term effect.

Wubbels and Korthagen (1990) report control analyses for potential effects on the results presented above of the types of classes and the type of schools included in the sample. No effects were found and therefore they conclude that the graduates of the Utrecht programme are perceived by their pupils to show behaviours that are better for pupil outcomes than graduates of the more traditional programme. Thus in this respect graduates from the Utrecht programme may be considered better teachers.

### Quality of the teacher self-perception

The quality of a teacher’s perception of his or her own interpersonal behaviour was assessed on the basis of the conformity between that teacher
perception and the perception which pupils have of this relationship. The comparison of teacher and pupils’ perceptions was carried out by calculating for each class of a particular teacher the sum (for the eight scales) of the absolute difference between the average pupil score in that class and the teacher score. Then the averages of these absolute difference scores in the two groups were compared using a $t$-test. The absolute difference score was 3.4 in the experimental group and 3.8 in the control group. These scores do not differ significantly ($t = 1.5$ and $p = 0.13$). Nor was a significant difference registered between the two groups in the case of teachers who graduated one or two years before. Among those who graduated more than two years before, however, there is a significant difference: the discrepancy between the teacher and pupil perceptions of the teacher behaviour is smaller in the Utrecht group than in the other group (scores were 3.1 and 3.8 respectively, $t = 2.2$ and $p = 0.02$, the effect size is about 0.5 standard deviation). We conclude that the teachers who graduated more than two years before from the Utrecht programme, have a more appropriate self-perception than the teachers in the control group with more than two years experience. This seems to be an indication that, in the long run, the Utrecht teachers have a higher level of reflectivity with regard to their own teaching.

6. DISCUSSION

We conclude that the teacher education programme was successful in changing the student teachers’ views of mathematics education, especially in the direction of a more inquiry oriented approach, and in promoting effective teacher behaviour in the classroom.

In this section we focus on the less successful aspects of the programme, especially on the fact that only a small number of the student teachers seemed to recognize and use the principle of building on pupils’ own constructions, which is an important feature of realistic mathematics education. How can we explain this lack of success of the programme and what could we learn from it, also for other mathematics teacher education programmes?

6.1. The nature of teachers’ conceptions

First, one central problem is that any attempt to change conceptions about a certain subject domain or conceptions about teaching is always a difficult enterprise (Wahl et al. 1984; Turk & Speers, 1983), especially if these conceptions are rooted in life-long experiences as pupils (Kagan, 1992;
Stereotyped examples of teaching shown by film and on television may contribute to the stability of student teachers’ conceptions of teaching (Lasley, 1980).

One of the major causes for the difficulty of changing teachers’ conceptions may be that teacher education programmes disregard the non-rational, intuitive and holistic nature of these conceptions (Wubbels, 1992). Most teacher educators try to influence student teachers by means of rational, analytical ways of dealing with these conceptions (see Korthagen, 1993, for a discussion of the reasons for this), but the impact of such approaches on the student teachers’ conceptions is relatively low.

One way to improve teacher education programmes such as the one described in this article, would be to include programme elements in which student teachers’ non-rational, intuitive images or ‘gestalts’ (Huibregtse, Korthagen & Wubbels, 1994) are dealt with directly. Korthagen (1993) describes some unconventional techniques such as making drawings or bringing old images into consciousness through guided fantasies. Another technique that could be helpful in teacher education is the so-called ‘metaphor technique’, which we used recently in our research with the aim to make teachers’ implicit views of mathematics education explicit. In this approach, similar to the work of Weber and Mitchell (1995), the teacher chooses one picture from a set of pictures (see for examples Figure 4). The chosen picture should be representative for his or her image of mathematics education. During a special procedure, the deeply engrained notions, feelings, values, attitudes and behavioural tendencies related to the teacher’s image are brought into awareness (see for a detailed description of the procedure Dolk, Korthagen & Wubbels, 1995).

6.2. Insufficient practice

The Utrecht program emphasized strongly the principles of realistic mathematics education both in the mathematics content and in the process of the course. As one reviewer of the first draft of this paper suggested, the students may have experienced realistic mathematics as students but not sufficiently as managers of learning and instruction. They may not have internalized satisfactorily the principles of realistic mathematics education as a framework for their work as a teacher. The programme of course deals with the teacher management of the curriculum in high school, but during the years that this study was conducted, realistic mathematics education was only partly introduced in the schools. Many schools in which the student teachers had their teaching practice still worked in a more traditional way. Therefore, a large part of the programme necessarily has to deal with the students’ own learning of realistic mathematics. Thus they may have
Figure 4. Metaphors for teaching
experienced to a sufficient degree the philosophy of realistic mathematics education but may have not been sufficiently exposed to tasks to develop realistic mathematics education for pupils.

In line with the work by Van Dormolen (1986), we propose textual analysis of the high school mathematics text books as one of the means to help students conceptualize the school curriculum in a way that is more in line with the principles of realistic mathematics education. In the common Dutch schools the materials (mostly textbooks and books with exercises) govern the daily teaching practice. When analyzing a textbook many student teachers only recognize the mathematical content (algorithms, concepts and structures) and not the more difficult to describe ‘translation skills’ and other competencies that play an important role in realistic mathematics. Therefore, the student teachers’ scope needs to be broadened for example with textual analysis of both traditional and realistic text books. Textual analysis can be used as a strategy to help prospective teachers to learn to take into consideration more aspects of teaching and learning. As a result of that their way of using texts as a help for learning might change.

The textual analysis can concentrate on focal points that are pragmatic rather than philosophical. These are points that would evolve from almost any reasonable reflection about what ought to be contained in a school mathematics textbook, such as:

- correctness of the content
- adaptation to pupils’ abilities
- preparation of the pupils for the text by what they have already learned and the preparation for what they will have to learn in the future

Because of the background of the student teachers not all of the focal points used will be obvious for them. The most important focus point for the shift to realistic matnematics education is the difference between problem situation and mathematical kernel mentioned by Van Dormolen (1986). ‘Mathematics is not human activity and cultural heritage alone: there must be a reason for the activity, a problem situation from which, by means of activity, new knowledge arises such as theorems, rules, definitions, methods, conventions. We shall indicate such elements of this knowledge by kernels. The nature of the activity, e.g. generalizing, formalizing, applying, patterning, ordering etc., depends on the nature of the problem situation which can be either mathematical or non-mathematical’ (Van Dormolen, 1986, p. 145, 146). At the background of these ideas of Van Dormolen is the conviction that the learning processes for mathematical concepts and competencies are highly content specific. A teacher has to decide first what aspect of the kernel might be recognizable to his pupils from the text on the basis of an analysis of the pupils’ existing cognitions. Then the teacher
has to decide what additional learning experiences he or she would like his pupils to have, besides those gained from reading the text. The teacher must be able to see what aspects of a certain kernel are lacking in the text and he or she has to decide if this deficiency should be remedied and if so, how. At the same time the teacher has to be aware of the problems that may arise from the use of a special context. In conclusion, textual analysis requires and promotes the development of many competencies that are essential in the context of ‘building on pupils’ own constructions’. As such, textual analysis may help to bridge the gap between conceptions about teaching mathematics, development in the programme and practice.

6.3. Sudden immersion

Another explanation for the relatively low impact of the programme on the aspect of building on pupils’ own constructions could be that the students in the programme are thrown too much in the deep end. Student teachers are confronted from the first day with an approach that differs largely from their experiences at high school and thus they may become reluctant to follow their teacher educators. The tensions connected to be required to teach whole lessons in student teaching from the very beginning may have blocked their development. In the realistic (and constructivist) view of learning it is very important that teachers can build on the knowledge and way of learning of their pupils. Because of that it is essential for a teacher who teaches according to the realistic approach, to be able to listen to the pupils in order to understand their conceptions and to be able to get feedback from these pupils (Broekman, 1990). However, during teaching practice it appears that the student teacher often loses sight of the individual pupil because of classroom management problems and especially because of that which most new teachers fear: discipline problems.

It might be helpful to introduce the realistic conception more gradually, thus allowing more time to build on the student teachers’ preconceptions. Such gradualness probably can be realized for example by introducing teaching practice via real but simplified situations. The prospective teacher can work with only one pupil for one hour a week for about 6 or 7 weeks. This one-to-one teaching experience may enable student teachers to gain experience in reflection, stimulated by using a logbook, audio recordings of the one-to-one lessons and discussions with fellow student teachers and the supervisor (see Vedder & Bannink, 1988). In this approach the trainees may discover in their reflection after their first one-to-one lesson how much time they have been talking and explaining things to the pupil. An audio recording often makes clear that most of the explanations did not really work for example because one can hear on the tape that the pupil
is desperately trying to ‘repeat’ what the student teacher said before (this phenomenon is also described by Van Hiele, 1986, p. 110).

6.4. **Concluding remarks**

The research presented in this article shows that the preparation of teachers for realistic mathematics education is no simple task. It requires the search for techniques and strategies in teacher education which induce important changes in the student teachers’ conceptions of mathematics and mathematics teaching, conceptions which are deeply engrained. We consider the fact that student teachers are confronted with more traditional approaches towards mathematics education, both as pupils and in their practical training in the schools, as a major factor making it difficult to realize the desired changes. Moreover, the current teacher education programmes may underestimate the impact of non-rational, intuitive images of teaching.

The fact that during the last decade, both Dutch textbooks and examinations in mathematics have changed considerably, may help to break this vicious teaching-learning-teaching cycle. On the other hand, how ‘realistic’ the textbooks or the examinations may be, a principle such as building on pupils’ own constructions, is to be realized by the teacher and can only come to the fore through his or her actual way of teaching. We hope to have offered ideas for influencing this aspect of mathematics education, the teacher’s classroom behaviour, which, in our opinion, lies at the very heart of mathematics education.

7. **APPENDIX 1**

7.1. **Examples of problems from the mathematics curriculum in the teacher education program (1st year).**

**Problem 1**

The sensitivity of photographic film can be expressed in terms of DIN or ASA:

- DIN: 15 18 20 21 22 24 30
- ASA: 25 50 80 200 125 200 800

Can you find a formula which describes the relationship between DIN and ASA?

**Comment**

This is one of many problems in the curriculum which originate from an everyday context, and which can be solved with high school mathematics.
Thus it is an example of ‘using the mathematics you know’. However, quite often students do not find it that simple to do. In the program, the methods used in working on such problems get more attention than the solution itself. Theories about problem-solving are used as frames of reference in the process of analyzing one’s own mathematical activities.

Problem 2

*How far can you see when you are standing 20 meters above the earth’s surface? Go up to the 4th floor, or to a high building in the surroundings, and check your answer.*

**Comment**

This is another example of the use of mathematics in everyday life. Important aspects are the construction of a model (the earth as a sphere), and the comparison of the result, acquired by means of this model, with reality. (Close to the earth, surface aberrations play a role.) This leads to reflection on the use of mathematical models in general.

Problem 3

*Can you make a reasonable guess at the number of dimes which you could put on this page (without overlapping)? You can also try to actually find out how many dimes you can put on this page. How do you know no greater number is possible?*

**Comment**

The student teacher first has to decide (in the smaller group) what a reasonable guess is. A useful heuristic approach would be to simplify the problem by using a smaller piece of paper. The last sentence forces the students to reflect on their reasoning.

Problem 4

*What direction is Moscow?*

**Comment**

In this problem one of the main difficulties is discovering the relevant aspects, such as the direction of Moscow on the map and the which way is north. Once, a group of student teachers put arrows in the front yard of the college which pointed to several cities. Note that the arrows all pointed to
the ground!

Problem 5

Ship P is going in a straight line toward point B on the shore, at a constant speed. Ship Q is going in the direction of A at double the speed of ship P. How close do the two ships get?

Comment

This is a problem which allows for many different approaches, ranging from drawing and using a ruler (hands-on activities!), to advanced solutions based on the use of vectors.

REFERENCES


*IVLOS Institute of Education*
*Utrecht University*
*P.O. Box 80127*
*3508 TC Utrecht*
*The Netherlands*